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Adversarial Risk Analysis: The Somali Pirates Case

Juan Carlos Sevillano
Universidad Complutense de Madrid, 28040 Madrid, Spain, sevimjc@ccee.ucm.es
David Rios Insua
Royal Academy of Sciences, 28004 Madrid, Spain, david.rios@urjc.es
Jesus Rios
IBM T.J. Watson Research Center, Yorktown Heights, New York 10598, jriosal@us.ibm.com

We show how adversarial risk analysis may cope with a current important security issue in relation with piracy off the Somali coasts. Specifically, we describe how to support the owner of a ship in managing risks from piracy in that area. We illustrate how a sequential defend–attack–defend model can be used to formulate this decision problem and solve it for the ship owner. Our formulation models the pirates' behavior through the analysis of how they could solve their decision problem.

Key words: applications: security; adversarial risk analysis; defender–attacker models; piracy; practice
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1. Introduction

As described by Lomborg (2008), several of the world's biggest problems revolve around security issues. These include terrorism, conflicts, arms proliferation, money laundering, and organized crime. Indeed, multibillion-euro investments are being made globally to increase security, which has stirred public debate about the costs and benefits of such measures, specially in a shrinking economy. This has motivated a great deal of interest in security resource allocation models, with varied techniques, e.g., those of Gutfraind (2009), Wein (2009), Parnell et al. (2008), Enders and Sandler (2009), and Devlin and Lorden (2007). Their key feature is the presence of two or more intelligent opponents who make decisions whose outcomes are interdependent and uncertain. Many of those models combine risk analysis, decision analysis, and game theory, as reviewed by Merrick and Parnell (2011), who comment favorably about adversarial risk analysis (ARA), introduced by Rios Insua et al. (2009). In supporting one of the participants, ARA views the security resource allocation problem as a decision analytic one, but procedures that employ the game theoretical structure, and other available information, are used to estimate the probabilities of the opponents' actions.

In Rios and Rios Insua (2012), ARA is used to analyze three important counterterrorism resource allocation models and is illustrated with simple numerical examples. In this paper, we consider how ARA may cope with a realistic example, referring to defending a ship that needs to travel through the Aden Gulf while facing piracy risks. Starting from the early 1990s, piracy has been a threat to international marine transportation and fishing ships around the coasts of Somalia.[1] Since 2005, several international organizations have expressed their worries about the increase in piracy acts. Nowadays, no ship is safe within several hundred miles of the Somali coast, and this has become a major international security issue.

Somali pirates, originally dedicated to fishing, have traditionally claimed that the actual pirates are the foreign fishermen who loot their fish. Piracy in Somalia may be explained purely in business terms (see Carney 2009), because there is actually a whole system

[1] See the Wikipedia page on piracy in Somalia: http://en.wikipedia.org/wiki/Piracy_in_Somalia (accessed March 6, 2012).

supporting their activities. The elderly act, de facto, as a government. Local businessmen provide funding. There is a clear organization, because attacks are undertaken by small groups of about 10 individuals in fast offshore boats that depart from a mother ship. Once successful, approximately 50 pirates remain in the boarded ship, with around approximately 50 more pirates providing logistic support from the coast. Pirates have learned that ransom is more profitable than theft, and they reinvest part of their earnings in equipment and training.

We shall assume that we support a (large tuna fish) ship owner in deciding what defensive resources to implement and, if attacked and hijacked, how to respond to Somali pirates demanding a ransom in exchange for the kidnapped ship and crew. Our intention is twofold. On one hand, we illustrate ARA on a realistic problem; on the other hand, the case study may serve as a template for future ARA applications.

2. Structuring the Somali Pirates Case

We describe here how to support a ship owner in managing risks from piracy along the coast of Somalia, structuring the problem through a game tree. We shall use a sequential defend–attack–defend model (see Brown et al. 2006, Parnell et al. 2010, Rios and Rios Insua 2012) to formulate the Somali pirates case. The ship owner will proactively decide on a defensive strategy to reduce piracy risks, ranging from different levels of deployed armed security to sailing via an alternative, much longer, route avoiding the pirates completely. The pirates, who have a network of spies and observers informing about security, cargo, and crew in ships, will respond to the defender's move by launching (or not) an attack with the intention of taking over the ship and asking for a ransom. If the pirates' operation is successful, the ship owner will have to decide on paying the ransom or not, or even asking for the support of the nearby armed forces to release the ship. The ship is assumed to be of Spanish ownership, with Spanish crew, for the associated financial computations.

Specifically, we shall assume that the ship owner (the defender, "she") initially decides on one of the following four alternative defense actions (elements of $\mathcal{D}_1$):

- d_1^0: do nothing, i.e., no defensive action is taken;
- d_1^1: use private protection with an armed person;
- d_1^2: use private protection with a team of two armed persons; or
- d_1^3: do the trip through the Cape of Good Hope rather than the Suez Canal, thus avoiding the Somali coast and a potential hijack.

Once the defender has made her initial decision, the pirates (the attacker, "he") observe this and decide whether to attack ($a^1 \in \mathcal{A}$) or not ($a^0 \in \mathcal{A}$). The attack results in either the ship being hijacked ($S = 1$) or not ($S = 0$) by the pirates, with probabilities depending on the initial defense action chosen by the ship's owner. If the ship is hijacked, the defender has the option of responding by any of the following (elements of $\mathcal{D}_2$):

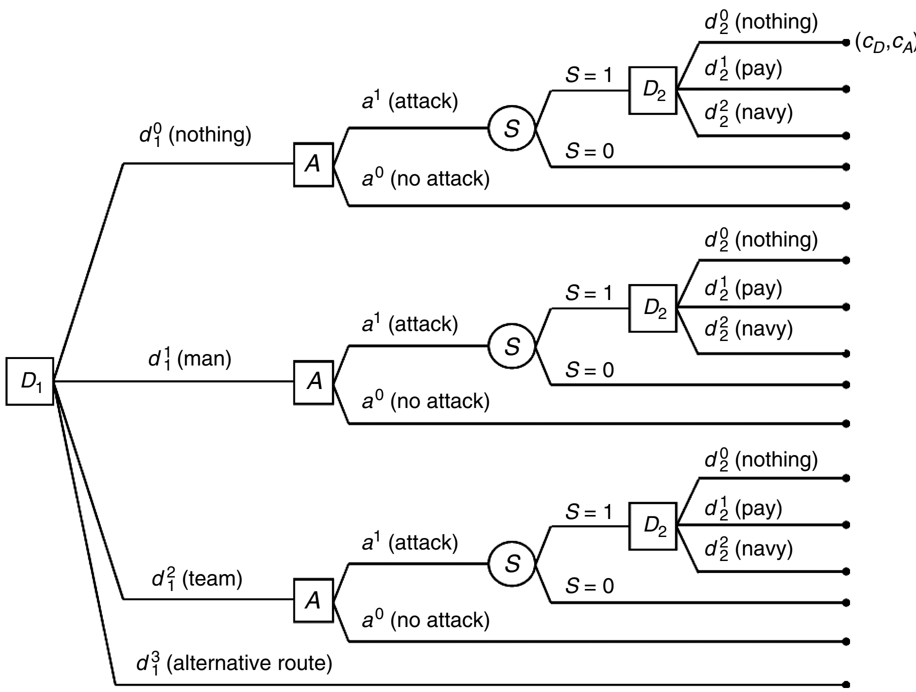
- d_2^0: doing nothing, i.e., not responding to the pirates' demands, assuming all entailed costs;
- d_2^1: paying the amount finally demanded by the pirates, thus recovering the ship and crew;
- d_2^2: asking for naval military support to release the ship and crew.

The asymmetric game tree shown in Figure 1 represents the sequence of decisions and events faced by the owner and pirates in this case, where nodes D_1 and D_2 correspond, respectively, to the defender's first and second decisions, node A represents the attacker's decision, and chance node S represents the outcome of the attack. We include (c_D, c_A), which represent the generic consequences that the defender and the attacker face, respectively, for the corresponding sequence of decisions and attack results.

3. Modeling the Defender's Own Preferences and Beliefs

In supporting the defender, her decision problem is seen as the decision tree in Figure 2, in which the attacker's decision node A has been replaced by chance node a. This reflects that the attacker's decision is seen as an uncertainty by the defender, with the bulk of the modeling work consisting of the assessment of her probabilities over the attacker's actions. Thus, to solve her decision problem, she needs to assess $p_D(A \mid d_1)$, her predictive probability of an attack given each $d_1 \in \mathcal{D}_1$, besides the more standard assessments $p_D(S \mid d_1, a^1)$ and $u_D(c_D)$, with c_D representing her monetary cost equivalent of the multiattribute consequences associated with each leaf of the tree. We now specify the standard assessments, starting with her preferences.

Figure 1 Game Tree for the Somali Pirates Case



The relevant consequences for the defender in this case are

- the loss of the ship,
- the costs associated with defending and responding to an eventual attack, and
- the number of deaths on her side.

As far as the costs associated with implementing her protective and response actions against an eventual hijacking are concerned, for the defensive actions in $\mathcal{D}_1$, we have the following:

- 0 euros if she chooses to do nothing, d_1^0;
- 0.05 million euros, if she chooses to use one armed person, d_1^1 (this corresponds to the salary of the armed person for six months, plus equipment);
- 0.15 million euros, if she chooses an armed team, d_1^2 (this cost corresponds to the salary of two armed persons, with better equipment, for six months);
- 0.5 million euros, if she chooses d_1^3, going around the Cape of Good Hope (cost is due to the longer distance of the trip; bad weather conditions might also increase the cost of this longer trip, but we shall neglect this uncertainty here).

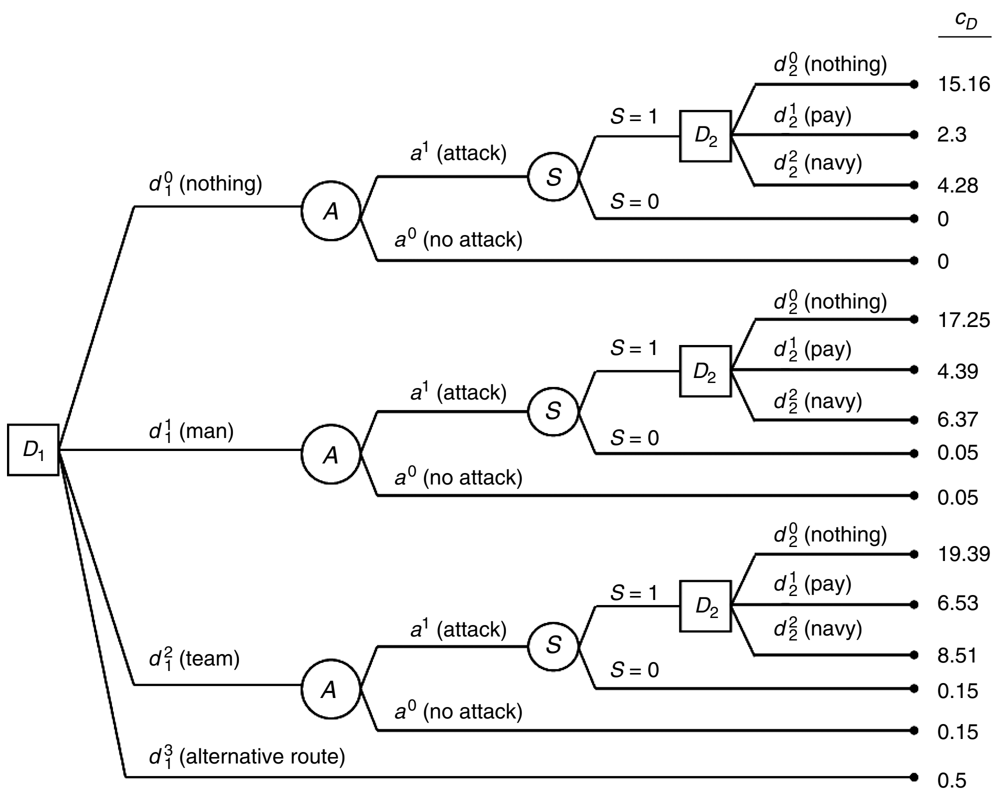
The costs associated with the defense actions in $\mathcal{D}_2$ are as follows:

- 0 euros is the cost for option d_2^0, doing nothing.
- 2.3 million euros is the cost for option d_2^1, paying the ransom. We have estimated it through the average of the latest ransoms paid, which were 2.2 million for *Le Ponant*, 2 million for the MT *Stolt Melati* 5, 1.1 million for the MT *Stolt Valor*, 3 million for the *Sirius Star*, and 3.2 million for the MV *Faina* (see Carney 2009). Given the uncertainty in the ransom negotiations involved, we could model this as a random variable, but, for simplicity, we shall not do this here.
- 0.2 million euros is the cost for option d_2^2, calling for the navy. This estimation is based on a military intervention using the international coalition ships already deployed in the area, including one Spanish ship.

As far as human lives are concerned, we consider that if the ship is attacked and the attack is aborted ($S = 0$), there are no lives lost. If the attack is successful ($S = 1$), we assume that the armed defenders have died and that, depending on the chosen response at D_2, there might be additional lives lost. Specifically:

- If the response to the kidnapping is doing nothing, d_2^0, the pirates might kill part of the crew as a

Figure 2 Decision Tree Representing the Defender's Decision Problem for the Somali Pirates Case



warning for future hijacks. We estimate this to be four crew members, though again some uncertainty exists, which we do not treat here.

- If the ship owner decides to pay the ransom, d_2^1, there will be no additional human loss.
- If the hijacked ship is rescued by the navy, d_2^2, we estimate that there might be two casualties due to both collateral damage during the intervention and/or because the pirates feel threatened and kill some of the crew during the operation. Again, there would be uncertainty involved, not treated here.

We monetize the value of the ship and crew lives. The types of ships operating in that area, focusing on tuna fishing, have a length between 80 and 110 m and a lot of technology built in. A new ship of this class costs between 9 and 12 million euros. Assuming some depreciation because of time, we shall consider that the incumbent ship is valued at 7 million euros. As far as quantifying the value of a human life, we shall use the concept of statistical value of a life (see Martinez et al. 2009), which, according to Riera et al. (2007), was estimated as 2.04 million euros for a Spanish person.

Table 1 summarizes the estimated consequences and aggregated monetary costs c_D for the defender associated with each scenario consisting of a path in the tree shown in Figure 1. Clearly, if there is no attack ($a = a^0$), we have that $S = 0$.

We shall assume that the defender is constantly risk averse with respect to monetary costs. Thus, her utility function is (strategically equivalent to) $u_D(c_D) = -\exp(c \times c_D)$, with $c > 0$. We shall study what happens when $c \in \{0.1, 0.4, 1, 2, 5\}$ as a sensitivity analysis.

Based on information from Carney (2009), we shall assume that the defender's beliefs about an attack being successful conditional on no initial defensive action taken would be $p_D(S = 1 \mid a^1, d_1^0) = 0.40$. We shall also assume that

- $p_D(S = 1 \mid a^1, d_1^1) = 0.10$ for the case in which she uses private protection with an armed person, and

Table 1 Consequences of Various Tree Paths for the Defender

D_1	S	D_2	Ship loss	Action costs	Lives lost	c_D
d_1^0 (nothing)	$S=1$	d_2^0 (nothing)	1	$0+0$	$0+4$	15.16
d_1^0 (nothing)	$S=1$	d_2^1 (pay)	0	$0+2.3$ M	$0+0$	2.3
d_1^0 (nothing)	$S=1$	d_2^2 (navy)	0	$0+0.2$ M	$0+2$	4.28
d_1^0 (nothing)	$S=0$		0	0	0	0
d_1^1 (man)	$S=1$	d_2^0 (nothing)	1	0.05 M $+0$	$1+4$	17.25
d_1^1 (man)	$S=1$	d_2^1 (pay)	0	0.05 M $+$ 2.3 M	$1+0$	4.39
d_1^1 (man)	$S=1$	d_2^2 (navy)	0	0.05 M $+$ 0.2 M	$1+2$	6.37
d_1^1 (man)	$S=0$		0	0.05 M	0	0.05
d_1^2 (team)	$S=1$	d_2^0 (nothing)	1	0.15 M $+0$	$2+4$	19.39
d_1^2 (team)	$S=1$	d_2^1 (pay)	0	0.15 M $+$ 2.3 M	$2+0$	6.53
d_1^2 (team)	$S=1$	d_2^2 (navy)	0	0.15 M $+$ 0.2 M	$2+2$	8.51
d_1^2 (team)	$S=0$		0	0.15 M	0	0.15
d_1^3 (alternative route)			0	0.5 M	0	0.5

- $p_D(S=1 \mid a^1, d_1^2) = 0.05$ for the case in which she uses private protection with two armed persons,

and we shall check sensitivity with respect to such assessments making them smaller (0.05 and 0.025, respectively) and higher (0.2 and 0.1, respectively).

4. Modeling the Defender's Beliefs Over the Attacker's Actions

We describe now how the defender may estimate the probability of being attacked, given her implemented initial defense. Carney (2009) suggests that the probability of being attacked is 0.005 based on historical data on piracy off the coast of Somalia. However, this estimate does not take into account that some ships may be more desirable than others (in terms of obtaining a bigger ransom) for the pirates who typically use observers and informers to decide upon their targets.

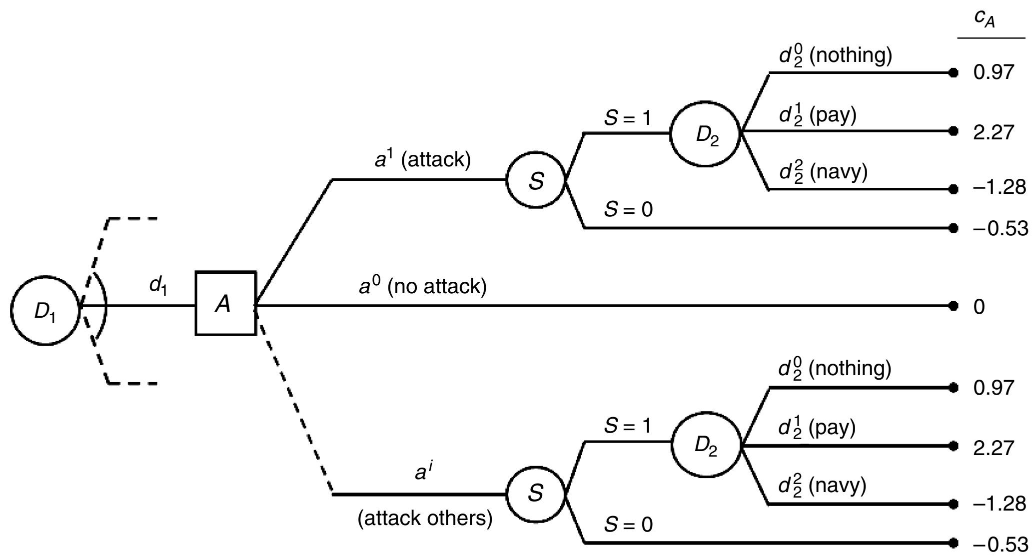
Our objective is to estimate a predictive probability of attack $p_D(A = a^1 \mid d_1)$ for the type of ship owned by the defender, conditional on each possible initial protective defense $d_1 \in \mathcal{D}_1 \backslash \{d_1^3\}$ taken. Instead of estimating these probabilities based only on data, as in Carney (2009), which corresponds to a zero-level analysis, we do so based on a one-level analysis (see Banks et al. 2011 and the discussion by Kadane 2011). To do so, we assume the attacker behaves as an expected utility maximizer and derive the defender's uncertainty about the attacker's decision from her uncertainty about the attacker's probabilities and utilities. Thus, the defender must analyze the decision problem faced by the attacker from her perspective, as shown in Figure 3. Note that the set of alternatives for the attacker has been expanded to include alternatives $a^i \in \mathcal{A}$, for $i = 2, \ldots, n$, representing the pirates' option to attack other ships that are not owned by our defender. We have also added new chance nodes D_2 at the end of the tree paths starting at a^i, representing the responses of ships $i = 2, \ldots, n$ to an eventual kidnapping, which are considered uncertainties from the perspective of the attacker. This analysis of the pirates' decision entails the probabilistic assessment of the pirates' perceived preferences (the uncertainty of the defender over the attacker's preferences, modeled through the random variables $U_A(a, s, d_2)$, where now $a \in \mathcal{A} = \{a^0, a^1, \ldots, a^n\}$) and beliefs (the uncertainty of the defender over the attacker's beliefs, modeled through the random variables $P_A(S=1 \mid a^1, d_1)$ and $P_A(D_2 \mid d_1, a^1, S=1)$ as well as $P_A(S=1 \mid a^i)$ and $P_A(D_2 \mid a^i, S=1)$ for $i = 2, \ldots, n$). For simplicity, we shall assume that the n ships are of similar value and features, but see our comments below.

The defender considers that the relevant consequences for the pirates are

- whether they keep the ship or not,
- the amount of money earned, and
- the number of pirates' lives lost.

As described by Carney (2009), the estimated average cost of an attack operation is approximately 30,000 euros (0.03 million euros). The eventual benefits if the defender pays the ransom are the above-mentioned 2.3 million euros. As far as human lives are concerned, we shall assume that two pirates are dead if the attack is repelled, and no pirates' lives are lost in the attack if successful. However, if the defender

Figure 3 Decision Tree Representing the Perceived Decision Problem of the Somali Pirates



responds by sending the navy, we shall assume that five pirates will be killed. Again, these numbers (ransom, lives lost, and, to a lesser extent, operational costs) would be affected by uncertainty, but we shall neglect it.

If the pirates keep the ship, it will not have the same monetary value as for the defender. They may use its machinery or its technological instruments, or they could use it as a mother ship. We shall assess its economic value for the pirates as 1 million euros. We shall assume that they put a value equivalent of 0.25 million euros to a pirate's life. Table 2 summarizes the consequences for the pirates of various attack scenarios, including the aggregate monetary equivalent c_A in the last column. As stated above, we have assumed that, for the pirates, there are no differences between the consequences of attacking our defender's ship (a^1) and those from attacking other ships ($a^i, i = 2, \ldots, n$), essentially implying that the n ships are of a similar type.

At a qualitative level, the defender thinks that the pirates are risk seeking over profits. Specifically, she assumes they are constantly risk seeking. Therefore, she uses a utility function (strategically equivalent to) $u_A(c_A) = \exp(c \times c_A)$, with $c > 0$, to model the pirates' preferences and risk attitudes. However, she is not sure about which c determines the pirates' utility function, but she thinks that $c \sim \mathcal{U}(0, 20)$. This uncertainty over c induces uncertainty over u_A to provide U_A.

Because the pirates most likely have access to the same information as the defender, she assesses the following Beta distributions for the pirates' beliefs over an attack on her ship being successful, conditional on her initial defense move:

- $P_A(S = 1 \mid a^1, d_1^0) \sim \mathcal{B}e(40, 60)$ for no defensive action taken,
- $P_A(S = 1 \mid a^1, d_1^1) \sim \mathcal{B}e(10, 90)$ for private protection with an armed person, and

Table 2 Consequences for Pirates of Various Tree Paths of Their Decision Problem, $i = 1, \ldots, n$

A	S	D_2	Ship kept	Profit	Lives lost	c_A
a^0 (no attack)			0	0	0	0
a^i (attack)	$S = 1$	d_2^0 (nothing)	1	−0.03 M	0	0.97
a^i (attack)	$S = 1$	d_2^1 (pay rescue)	0	2.27 M	0	2.27
a^i (attack)	$S = 1$	d_2^2 (navy sent)	0	−0.03 M	5	−1.28
a^i (attack)	$S = 0$		0	−0.03 M	2	−0.53

- $P_A(S=1 \mid a^1, d_1^2) \sim \mathcal{B}e(50, 950)$ for private protection with a team of two armed persons.

Note that the expected values of these distributions correspond with the assessed probabilistic beliefs of the defender on the same uncertainty. Likewise, the defender assesses that the probabilities representing the pirates' beliefs of a successful attack on ships $i=2,\ldots,n$, which we assumed are of the same class as the defender's ship, are as follows:

- $P_A(S=1 \mid a^i) \sim \frac{1}{3}\mathcal{B}e(40,60) + \frac{1}{3}\mathcal{B}e(10,90) + \frac{1}{3}\mathcal{B}e(50,950)$, $i=2,\ldots,n$, acknowledging our lack of information about the defense actions undertaken by other ships and, therefore, about their vulnerability levels. Note that we assume these defensive actions are observed by the pirates, but not by the defender.

Now, the defender assesses how the pirates think she will respond to a successful attack. Specifically, she thinks that the pirates expect her to respond along the same lines of defense that she chose at the first stage. Thus, a tough deterring defense at her first move is expected to produce an eventual response of similar harshness. Therefore, the defender assesses the following Dirichlet distributions over d_2^0 (doing nothing), d_2^1 (pay), and d_2^2 (navy) $\in \mathcal{D}_2$, representing her beliefs on the attacker's probabilities p_A:

- $P_A(D_2 \mid d_1^0, A=a^1, S=1) \sim \text{Dir}(1,1,1)$. If $d_1 = d_1^0$ (doing nothing) and the ship is hijacked, any response in this case is perceived to be equally likely by the attacker.
- $P_A(D_2 \mid d_1^1, A=a^1, S=1) \sim \text{Dir}(0.1,4,6)$. If $d_1 = d_1^1$ (protect with an armed man) and the ship is hijacked, it is perceived that the attacker expects the defender to respond by doing something, with sending the navy more likely than paying the ransom.
- $P_A(D_2 \mid d_1^2, A=a^1, S=1) \sim \text{Dir}(0.1,1,10)$. If $d_1 = d_1^2$ (protect with an armed team) and the ship is hijacked, it is perceived that it would be even more likely for the attacker to believe that the defender will respond by sending the navy.

Finally, the defender assesses that

- $P_A(D_2 \mid A=a^i, S=1) \sim \frac{1}{3}\text{Dir}(1,1,1) + \frac{1}{3}\text{Dir}(0.1,4,6) + \frac{1}{3}\text{Dir}(0.1,1,10)$, for $i=2,\ldots,n$, suggesting our lack of information about the other ships' defensive type and, correspondingly, their responses. Note that we are acknowledging the defender's uncertainty about the type of defensive actions taken by these other ships and observed by the pirates.

Based on her above assessments, the defender may solve the perceived pirates' decision problem using backward induction over the decision tree in Figure 3, propagating the uncertainty of her assessed random preferences and beliefs (U_A, P_A) of the attacker as follows:

- Compute the random expected utilities associated with the pirates choosing a^1 conditional on each of her initial protective defenses $d_1 \in \mathcal{D}_1 \setminus \{d_1^3\}$:

$$\Psi_A(d_1, a^1) = P_A(S=1 \mid d_1, a^1) \cdot \left[\sum_{d_2 \in \mathcal{D}_2} U_A(a^1, S=1, d_2) P_A(D_2 = d_2 \mid d_1, a^1, S=1)\right] + P_A(S=0 \mid d_1, a^1) U_A(a^1, S=0).$$

- Compute the random expected utilities associated with the pirates choosing a^i for $i=2,\ldots,n$:

$$\Psi_A(a^i) = P_A(S=1 \mid a^i) \cdot \left[\sum_{d_2 \in \mathcal{D}_2} U_A(a^i, S=1, d_2) P_A(D_2 = d_2 \mid a^i, S=1)\right] + P_A(S=0 \mid a^i) U_A(a^i, S=0).$$

- Compute the defender's predictive probabilities of being attacked $(A=a^1)$ conditional on each of her initial defenses $d_1 \in \mathcal{D}_1 \setminus \{d_1^3\}$:

$$p_D(A=a^1 \mid d_1) = \Pr\left(\Psi_A(d_1, a^1) > \max\left\{U_A(a^0), \Psi_A(a^2), \ldots, \Psi_A(a^n)\right\}\right).$$

These probabilities can be approximated by Monte Carlo simulation by drawing a sample $\{(u_A^k, p_A^k)\}_{k=1}^N \sim (U_A, P_A)$ from the pirates' random utilities and probabilities assessed by the defender and solving for each draw the pirates' decision problem as before. This generates a sample of when the optimal decision for the pirates is $a_k^*(d_1) = a^1$, and then we approximate $p_D(A=a^1 \mid d_1)$ by

$$\frac{\#\{1 \le k \le N : \psi_A^k(d_1, a^1) > \max\{u_A^k(a^0), \psi_A^k(a^2), \ldots, \psi_A^k(a^n)\}\}}{N},$$

where # stands for the cardinality of a set.

For illustrative purposes, let us assume that $n=9$: there will be eight other ships (of similar class)

Table 3 Sensitivity Analysis

$p_D(S=1 \mid a^1, d_1)$			$\hat{p}_D(A=a^1 \mid d_1)$					
d_1^1	d_1^2	n	d_1^0	d_1^1	d_1^2	c	d_1^*	$d_2^*(d_1^*)$
0.2	0.1	5	0.41010	0.18354	0.00382	0.1 – 1 2 – 5	d_1^2 (team) d_1^3 (GH route)	d_2^1 (pay)
		9	0.27260	0.05780	0.00008	0.1 0.4 – 1 2 – 5	d_1^1 (man) d_1^2 (team) d_1^3 (GH route)	d_2^1 (pay) d_2^1 (pay)
		15	0.18322	0.01622	0.00000	0.1 – 0.4 1 – 5	d_1^1 (man) d_1^2 (team)	d_2^1 (pay) d_2^1 (pay)
		20	0.14230	0.00628	0.00000	0.1 – 1 2 – 5	d_1^1 (man) d_1^2 (team)	d_2^1 (pay) d_2^1 (pay)
0.10	0.05	5	0.46564	0.12166	0.00328	0.1 0.4 – 1 2 – 5	d_1^1 (man) d_1^2 (team) d_1^3 (GH route)	d_2^1 (pay) d_2^1 (pay)
		9	0.30332	0.02560	0.00004	0.1 – 0.4 1 – 2 5	d_1^1 (man) d_1^2 (team) d_1^3 (GH route)	d_2^1 (pay) d_2^1 (pay)
		15	0.19386	0.00392	0.00000	0.1 – 1 2 – 5	d_1^1 (man) d_1^2 (team)	d_2^1 (pay) d_2^1 (pay)
		20	0.14836	0.00098	0.00000	0.1 – 1 2 – 5	d_1^1 (man) d_1^2 (team)	d_2^1 (pay) d_2^1 (pay)
0.05	0.025	5	0.49010	0.09372	0.00374	0.1 0.4 – 1 2 – 5	d_1^1 (man) d_1^2 (team) d_1^3 (GH route)	d_2^1 (pay) d_2^1 (pay)
		9	0.31764	0.01596	0.00002	0.1 – 0.4 1 – 2 5	d_1^1 (man) d_1^2 (team) d_1^3 (GH route)	d_2^1 (pay) d_2^1 (pay)
		15	0.19842	0.00142	0.00000	0.1 – 1 2 – 5	d_1^1 (man) d_1^2 (team)	d_2^1 (pay) d_2^1 (pay)
		20	0.14778	0.00024	0.00000	0.1 – 1 2 – 5	d_1^1 (man) d_1^2 (team)	d_2^1 (pay) d_2^1 (pay)

Note. GH, Good Hope.

exposed to the risk of being seized by the pirates at the time period in which the defender's ship sails through the Gulf of Aden. Based on $N = 50{,}000$ Monte Carlo iterations, we get the following estimates for the probability of the defender's ship being attacked, given that she chooses as initial defense action $d_1 \in \mathcal{D}_1 \setminus \{d_1^3\}$:

- $\hat{p}_D(A = a^1 \mid d_1^0) = 0.30332$ for the case in which the defender does not take any protective action initially,
- $\hat{p}_D(A = a^1 \mid d_1^1) = 0.02560$ for the case in which she uses private protection with an armed person, and
- $\hat{p}_D(A = a^1 \mid d_1^2) = 0.00004$ for the case in which she uses private protection with a team of two armed persons.

Note that the probability of the defender's ship being attacked gets smaller if it is protected with an armed guard, and even smaller with an armed team. Also, these conditional attack probabilities would decrease as n gets bigger, as can be seen in Table 3. We can also analyze the impact of different vulnerabilities among ships if we elicit different values of $P_A(S=1 \mid a^i)$ for each type of ship, $i = 2, \ldots, n$. But we have assumed that they are all the same in this case.

5. Finding the Optimal Defense Strategy

We now have all the inputs needed to solve the decision problem for the defender. Given these, the defender can solve her decision problem working backwards in the tree in Figure 2. At decision node D_2, she can compute her maximum utility action conditional on each $d_1 \in \mathcal{D}_1 \backslash \{d_1^3\}$,

$$d_2^*(d_1, a^1, S=1) = \arg\max_{d_2 \in \mathcal{D}_2} u_D\big(c_D(d_1, S=1, d_2)\big).$$

Afterward, she will obtain at chance node S her expected utilities:

$$\begin{aligned}\psi_D(d_1, a^1) &= p_D(S=1 \mid d_1, a^1) u_D\big(c_D(d_1, S=1, d_2^*(d_1, a^1, S=1))\big) \\ &\quad + p_D(S=0 \mid d_1, a^1) u_D(c_D(d_1, S=0)).\end{aligned}$$

At this point, she will use her probabilistic assessments of being attacked conditional on her initial defense moves, $\hat{p}_D(A=a^1 \mid d_1)$, to compute for each $d_1 \in \mathcal{D}_1 \backslash \{d_1^3\}$ her expected utility at chance node A:

$$\begin{aligned}\psi_D(d_1) = \psi_D(d_1, a^1)\hat{p}_D(A=a^1 \mid d_1) \\ + u_D(c_D(d_1, S=0))(1 - \hat{p}_D(A=a^1 \mid d_1)).\end{aligned}$$

Finally, she can find her maximum expected utility decision at decision node D_1:

$$d_1^* = \arg\max_{d_1^i \in \mathcal{D}_1} \psi_D(d_1^i),$$

where $\psi_D(d_1^3) = u_D(c_D(d_1^3))$. Note that $c_D(d_1^3) = 0.5$ would be obtained from Table 1. The defender's best strategy is then to first choose d_1^* at node D_1 and, if the ship is attacked and hijacked, respond by choosing $d_2^*(d_1^*, a^1, S=1)$ at node D_2.

For each of the considered risk aversion coefficients determining her utility function, we obtain, based on the above estimates of $p_D(A=a^1 \mid d_1)$ for $n=9$, that the defense strategies of maximum expected utility are as follows:

- $c=0.1$ and $c=0.4$: protect with an armed man ($d_1^* = d_1^1$), and if hijacked ($S=1$), pay the ransom ($d_2^* = d_2^1$).
- $c=1$ and $c=2$: protect with a team of two armed men ($d_1^* = d_1^2$), and if hijacked ($S=1$), pay the ransom ($d_2^* = d_2^1$).
- $c=5$: avoid the Somali coast by going through the Cape of Good Hope ($d_1^* = d_1^3$).

We see that choosing to *go through the Cape of Good Hope* emerges as the optimal decision when the risk aversion coefficient of the defender is $c=5$, that is, when the defender becomes most risk averse. This suggests that some security measures are required, but if the defender is too risk averse it is better for her to change the route. The optimal defender's response action to a hijacking is always paying the ransom. This is possibly because it is the option that allows the defender to keep the ship and minimize the lives lost. Clearly, this neglects the political implications of this action, but recall that we are dealing with this problem from the ship owner's perspective. One possibility to acknowledge such a fact would be to add some extra cost if the ransom is actually paid, reflecting the negative political implication associated with it.

Finally, we show in Table 3 the maximum expected utility defense and estimated attack probabilities for several values of n and $p_D(S=1 \mid a^1, d_1)$, for $d_1 = d_1^1, d_1^2$, allowing us to check the sensitivity of the solution with respect to these quantities.

6. Conclusions

We have described an application of adversarial risk analysis to solve a Somali pirates case using a defend–attack–defend model. The program produced to solve this case may serve as a template for decision makers facing this same problem by just modifying the value of their ships, the value of their crew lives, the estimated ransom, their risk aversion coefficient, and other relevant inputs.

This analysis may also serve as a template for other security resource allocation problems adaptable to the sequential defend–attack–defend model. Essentially, the defender would:

1. build a game tree as in Figure 1;
2. assess her consequences as in Table 1 and the attacker's consequences as in Table 2;
3. assess her utility function, her vulnerabilities, and her beliefs over successful attacks as in §3;
4. assess her beliefs over the preferences and beliefs of the attacker as in §4;
5. (simulate to) estimate her conditional probabilities of various attacks as in §4;
6. solve her decision tree to find her optimal defenses as in §5 and perform a sensitivity analysis.

As we have commented, in some instances the consequences in step 2 might be random and described through random variables.[2] This would require one further random node in the decision tree solved in step 6 and one further loop in the Monte Carlo simulation addressed in step 5. Other problems might entail continuous defenses and/or attacks. That would typically entail solving nonlinear programming problems at decision nodes and discrete approximations at chance nodes. But the methodology would remain essentially the same.

As seen, the assessment of the probability of being attacked $p_D(A = a^1 \mid d_1)$ is straightforward as far as the defender is able to assess the random probabilities $P_A(D_2 \mid d_1, a^1, S = 1)$ and $P_A(D_2 \mid a^i, S = 1)$ for $i = 2, \ldots, n$. However, these assessments within a 1-level analysis could be problematic, as the defender may want to exploit information available to her about how the pirates analyze how she and other ship owners would respond to an eventual successful attack, which would correspond to a two-level analysis of the problem. This potentially may lead to an infinite regress, as described by Rios and Rios Insua (2012). However, realistically, we argue that this recursive analysis will always stop at some point when the defender has no more information that can be accommodated into the next level of analysis. At this point, the defender would use a noninformative distribution, as we have done for some of these assessments in our example.

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[2] Indeed, this would be the case in the example in this paper, but we have preferred to simplify this for clarity and because of space limitations.